Multiplication and division in polar form

Introduction

When two complex numbers are given in polar form it is particularly simple to multiply and divide them. This is an advantage of using the polar form.

1. Multiplication and division of complex numbers in polar form.

if $z_1 = r_1\angle\theta_1$ and $z_2 = r_2\angle\theta_2$ then

$$z_1 z_2 = r_1 r_2 \angle(\theta_1 + \theta_2), \qquad \frac{z_1}{z_2} = \frac{r_1}{r_2}\angle(\theta_1 - \theta_2)$$

Note that to multiply the two numbers we multiply their moduli and add their arguments.

To divide, we divide their moduli and subtract their arguments.

Example

If $z_1 = 5\angle(\pi/6)$, and $z_2 = 4\angle(-\pi/4)$ find a) $z_1 z_2$, b) $\dfrac{z_1}{z_2}$, c) $\dfrac{z_2}{z_1}$

Solution

a) To multiply the two complex numbers we multiply their moduli and add their arguments. Therefore

$$z_1 z_2 = 20\angle\left(\frac{\pi}{6} + \left(-\frac{\pi}{4}\right)\right) = 20\angle\left(-\frac{\pi}{12}\right)$$

b) To divide the two complex numbers we divide their moduli and subtract their arguments.

$$\frac{z_1}{z_2} = \frac{5}{4}\angle\left(\frac{\pi}{6} - \left(-\frac{\pi}{4}\right)\right) = \frac{5}{4}\angle\frac{5\pi}{12}$$

c)

$$\frac{z_2}{z_1} = \frac{4}{5}\angle\left(-\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{4}{5}\angle\left(-\frac{5\pi}{12}\right)$$

Exercises

1. If $z_1 = 7\angle\frac{\pi}{3}$ and $z_2 = 6\angle\frac{\pi}{2}$ find a) $z_1 z_2$, b) $\frac{z_1}{z_2}$, c) $\frac{z_2}{z_1}$, d) z_1^2, e) z_2^3.

Answers

1. a) $42\angle\frac{5\pi}{6}$, b) $\frac{7}{6}\angle -\frac{\pi}{6}$, c) $\frac{6}{7}\angle\frac{\pi}{6}$, d) $49\angle\frac{2\pi}{3}$, e) $216\angle\frac{3\pi}{2}$.