The straight line

Introduction

Straight line graphs arise in many engineering applications. This leaflet discusses the mathematical equation which describes a straight line and explains the terms 'gradient' and 'intercept'.

1. The equation of a straight line

Any equation of the form
$$y = mx + c$$
where m and c are fixed numbers, (i.e. constants), has a graph which is a straight line.

For example,
$$y = 3x + 5, \qquad y = \frac{2}{3}x + 8 \qquad \text{and} \qquad y = -3x - 7$$

all have graphs which are straight lines, but

$$y = 3x^2 + 4, \qquad y = \frac{2}{3x} - 7, \qquad \text{and} \qquad y = -14\sqrt{x}$$

have graphs which are not straight lines. The essential feature of a straight line equation is that x and y occur only to the power 1.

2. The straight line graph

Any straight line graph can be plotted very simply by finding just two points which lie on the line and joining them. It is a good idea to find a third point just as a check.

Example
Plot a graph of the straight line with equation $y = 5x + 4$.

Solution
From the equation, note that when $x = 0$, the value of y is 4. Similarly when $x = 3$, $y = 19$. So the points $(0, 4)$ and $(3, 19)$ lie on the graph. These points are plotted and joined together to form the straight line graph.
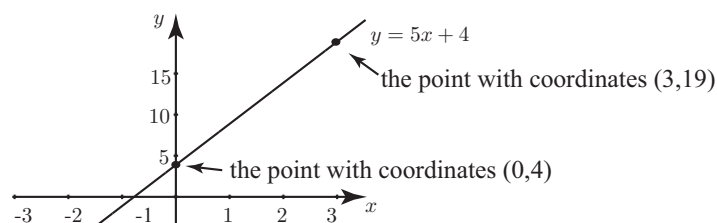
3. The gradient and intercept of a straight line

In the equation $y = mx + c$ the value of m is called the **gradient** of the line. It can be positive, negative or zero. Lines with a positive gradient slope upwards, from left to right. Lines with a negative gradient slope downwards from left to right. Lines with a zero gradient are horizontal.

this line has a positive gradient   this line has a negative gradient   the gradient of this line is zero
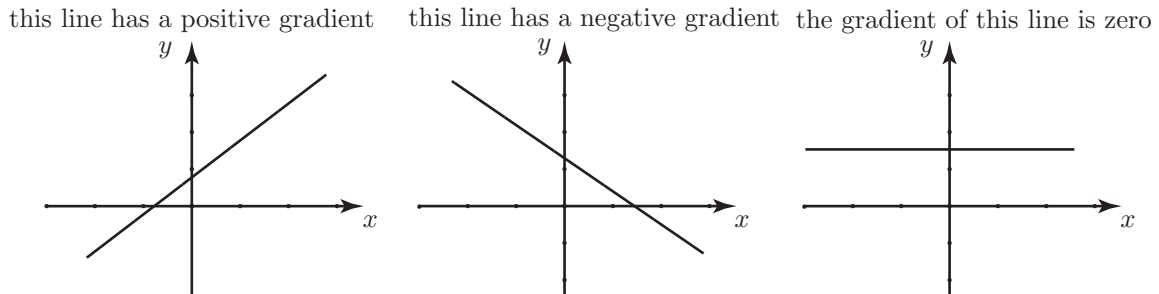
The value of c is called the **vertical intercept** of the line. It is the value of y when $x = 0$. When drawing a line, c gives the position where the line cuts the vertical axis.

the vertical intercept is positive

the vertical intercept is negative
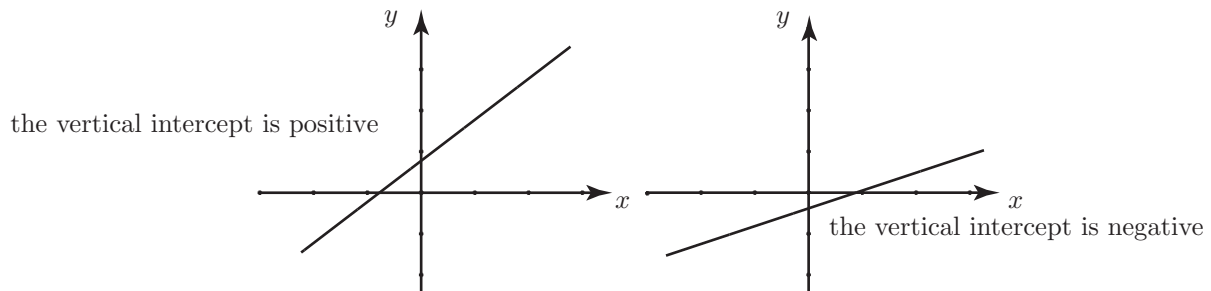
Example
Determine the gradient and vertical intercept of each line.

a) $y = 12x - 6$,   b) $y = 5 - 2x$,   c) $4x - y + 13 = 0$,   d) $y = 8$,   e) $y = 4x$.

Solution
a) Comparing $y = 12x - 6$ with $y = mx + c$ we see that $m = 12$, so the gradient of the line is 12. The fact that this is positive means that the line slopes upwards as we move from left to right. The vertical intercept is -6. This line cuts the vertical axis below the horizontal axis.

b) Comparing $y = 5 - 2x$ with $y = mx + c$ we see that $m = -2$, so the gradient is -2. The line slopes downwards as we move from left to right. The vertical intercept is 5.

c) We write $4x - y + 13 = 0$ in standard form as $y = 4x + 13$ and note that $m = 4$, $c = 13$.

d) Comparing $y = 8$ with $y = mx + c$ we see that $m = 0$ and $c = 8$. This line is horizontal.

e) Comparing $y = 4x$ with $y = mx + c$ we see that $m = 4$ and $c = 0$.

Exercises
1. State the gradient and intercept of each of the following lines.

a) $y = 5x + 6$,   b) $y = 3x - 11$,   c) $y = -2x + 7$,   d) $y = 9$,   e) $y = 7 - x$

Answers
1. a) gradient 5, intercept 6   b) 3,-11,   c) -2,7,   d) 0,9,   e) -1, 7.