Rearranging formulas 2

Introduction

This leaflet develops the work started on leaflet 2.11, and shows how more complicated formulas can be rearranged.

1. Further transposition

Remember that when you are trying to rearrange, or **transpose**, a formula, the following operations are allowed.

- add or subtract the same quantity to or from both sides
- multiply or divide both sides by the same quantity

A further group of operations is also permissible.

A formula remains balanced if we perform the same operation to both sides of it. For example, we can square both sides, we can square-root both sides. We can find the logarithm of both sides. Study the following examples.

Example
Transpose the formula $p = \sqrt{q}$ to make q the subject.

Solution
Here we need to obtain q on its own. To do this we must find a way of removing the square root sign. This can be achieved by squaring both sides since

$$(\sqrt{q})^2 = q$$

So,

$$\begin{aligned} p &= \sqrt{q} \\ p^2 &= q \qquad \text{by squaring both sides} \end{aligned}$$

Finally, $q = p^2$, and we have succeeded in making q the subject of the formula.

Example
Transpose $p = \sqrt{a+b}$ to make b the subject.

Solution

$$\begin{aligned} p &= \sqrt{a+b} \\ p^2 &= a+b \qquad \text{by squaring both sides} \\ p^2 - a &= b \end{aligned}$$

Finally, $b = p^2 - a$, and we have succeeded in making b the subject of the formula.

Example
Make x the subject of the formula $v = \frac{k}{\sqrt{x}}$.

Solution

$$\begin{aligned} v &= \frac{k}{\sqrt{x}} \\ v^2 &= \frac{k^2}{x} \qquad \text{by squaring both sides} \\ xv^2 &= k^2 \qquad \text{by multiplying both sides by } x \\ x &= \frac{k^2}{v^2} \qquad \text{by dividing both sides by } v^2 \end{aligned}$$

and we have succeeded in making x the subject of the formula.

Example
Transpose the formula $T = 2\pi\sqrt{\frac{\ell}{g}}$ for ℓ.

Solution
This must be carried out carefully, in stages, until we obtain ℓ on its own.

$$\begin{aligned} T &= 2\pi\sqrt{\frac{\ell}{g}} \\ \frac{T}{2\pi} &= \sqrt{\frac{\ell}{g}} \qquad \text{by dividing both sides by } 2\pi \\ \left(\frac{T}{2\pi}\right)^2 &= \frac{\ell}{g} \qquad \text{by squaring both sides} \\ \ell &= g\left(\frac{T}{2\pi}\right)^2 \end{aligned}$$

Exercises
1. Make r the subject of the formula $V = \frac{4}{3}\pi r^3$.
2. Make x the subject of the formula $y = 4 - x^2$.
3. Make s the subject of the formula $v^2 = u^2 + 2as$

Answers
1. $r = \sqrt[3]{\frac{3V}{4\pi}}$. 2. $x = \sqrt{4-y}$. 3. $s = \frac{v^2-u^2}{2a}$.